

# Electroweak Corrections to $e^+e^- \rightarrow \nu\bar{\nu}H$ and $e^+e^- \rightarrow t\bar{t}H^*$

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**Abstract.** The most interesting Higgs-production processes at future  $e^+e^-$  colliders belong to the process class  $e^+e^- \rightarrow f\bar{f}H$ . We study the full  $\mathcal{O}(\alpha)$  corrections to this reaction in the Standard Model for neutrinos and top quarks in the final state. Leading higher-order corrections from initial-state radiation and QCD corrections are also taken into account. Although cancellations between the different kinds of corrections occur, the full corrections are of the order of  $\pm 10\%$  and thus important ingredients in the theoretical predictions for future  $e^+e^-$  colliders.

**PACS.** 12.15.Lk Electroweak radiative corrections

## 1 Introduction

One of the most important future challenges in particle physics is the understanding of the mechanism of electroweak symmetry breaking and the discovery of the up to now only missing particle of the Electroweak Standard Model (SM), the Higgs boson. The mass of the Higgs boson is expected to be in the range between the current lower experimental bound of 114.4 GeV and 1 TeV, where a light Higgs boson (with  $M_H \sim 100\text{--}200$  GeV) is favoured by the global fit of the SM to electroweak precision data. While the LHC will find a SM Higgs boson in the full mass range up to 1 TeV if it exists and has no exotic properties, a complete determination of the Higgs interactions is only possible in the clean environment of an  $e^+e^-$  collider.

In this work we focus on the reactions  $e^+e^- \rightarrow f\bar{f}H$  where  $f$  is a neutrino or a top quark, which belong to the most interesting Higgs production processes at future  $e^+e^-$  colliders.

## 2 Radiative corrections to $e^+e^- \rightarrow \nu\bar{\nu}H$

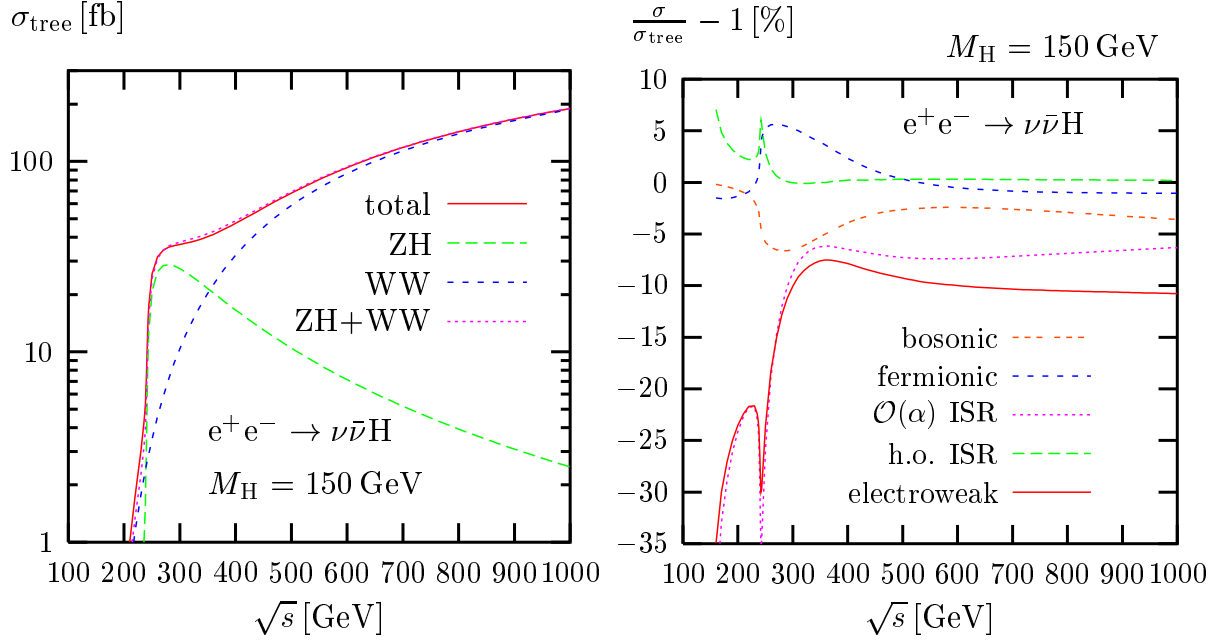
At an  $e^+e^-$  collider the two main Higgs production mechanisms in the SM are Higgs radiation off Z bosons, so-called Higgs strahlung, and Higgs production via WW fusion. Both mechanisms are present in the reaction  $e^+e^- \rightarrow \nu_l\bar{\nu}_lH$  where  $l$  can be an e,  $\mu$ , or  $\tau$ . The l.h.s. of Figure 1 shows the different contributions to the lowest-order cross section as a function of the centre-of-mass (CM) energy  $\sqrt{s}$  for  $M_H = 150$  GeV (see Ref. [1] for details). While

the Higgs-strahlung contribution to the total cross section rises sharply at threshold to a maximum of a few tens of GeV above  $\sqrt{s} = M_Z + M_H$  and falls off as  $1/s$ , the WW-fusion channel, which is only present in the reaction  $e^+e^- \rightarrow \nu_e\bar{\nu}_eH$ , dominates the cross section well above the ZH threshold and grows as  $\ln s$  in the high-energy limit. The difference between the total cross section and the sum ZH+WW is the interference between both channels which is relatively small.

The  $\mathcal{O}(\alpha)$  electroweak corrections to the process  $e^+e^- \rightarrow ZH$  have been calculated in Refs. [2,3,4], and a Monte Carlo algorithm for the calculation of the real photonic corrections to this process was described in Ref. [5]. The electroweak corrections to the full process  $e^+e^- \rightarrow \nu\bar{\nu}H$  have attracted a lot of interest recently. Analytical results for the one-loop corrections to this process have been studied in Ref. [6]; however, no numerical results are given there. The contributions of fermion and sfermion loops in the Minimal Supersymmetric Standard Model (MSSM) have been evaluated in Refs. [7,8]. Complete calculation of the full  $\mathcal{O}(\alpha)$  electroweak corrections to  $e^+e^- \rightarrow \nu\bar{\nu}H$  in the SM have been performed in Refs. [1,9]. These calculations agree within 0.3%, which is of the same order as the integration error of Ref. [9].

In the following we briefly summarize some results of our calculation [1] of the  $\mathcal{O}(\alpha)$  electroweak corrections, where we included also corrections from initial-state radiation (ISR) beyond  $\mathcal{O}(\alpha)$  in the structure-function approach. The calculation is done in the  $G_\mu$  scheme which absorbs the corrections proportional to  $m_t^2/M_W^2$  in the fermion-W-boson couplings and the running of  $\alpha(Q^2)$  from  $Q^2 = 0$  to the electroweak scale. The numerical evaluation of the virtual corrections is particularly complicated due to the appearance of pentagon diagrams. Therefore, we have applied the approach of Ref. [10] which avoids the ap-

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**Fig. 1.** Lowest-order cross section and contributions from ZH-production channel, WW-fusion channel, and their sum (l.h.s.) as well as relative corrections (r.h.s.) in the  $G_\mu$  scheme for a Higgs-boson mass  $M_H = 150$  GeV

pearance of inverse Gram determinants in the reduction of tensor 5-point functions. The soft and collinear singularities are treated both in the dipole subtraction method following Refs. [11,12] and in the phase-space slicing method following closely Ref. [13]. Two completely independent Monte Carlo programs have been constructed; one applies the multi-channel approach similar to Refs. [12,14,15], the second uses VEGAS [16].

The relative corrections are shown on the r.h.s. of Figure 1. The ISR corrections vary strongly in the region of the ZH threshold but are nearly flat for energies above 400 GeV. They are always negative since the lowest-order cross section is continuously rising. The fermionic corrections reach a maximum of about 6% in the region where the ZH-production channel dominates and are small above 500 GeV where the WW-fusion process is most important. The non-ISR bosonic corrections exhibit a minimum of about -7% at  $\sim 300$  GeV and are between 0% and -4% elsewhere. Near the ZH threshold the electroweak corrections are dominated by the  $\mathcal{O}(\alpha)$  ISR corrections from ZH production, while for higher energies, where the main contributions come from WW fusion, the corrections become flat reaching relative corrections of about -10% above 500 GeV.

### 3 Radiative corrections to $e^+e^- \rightarrow t\bar{t}H$

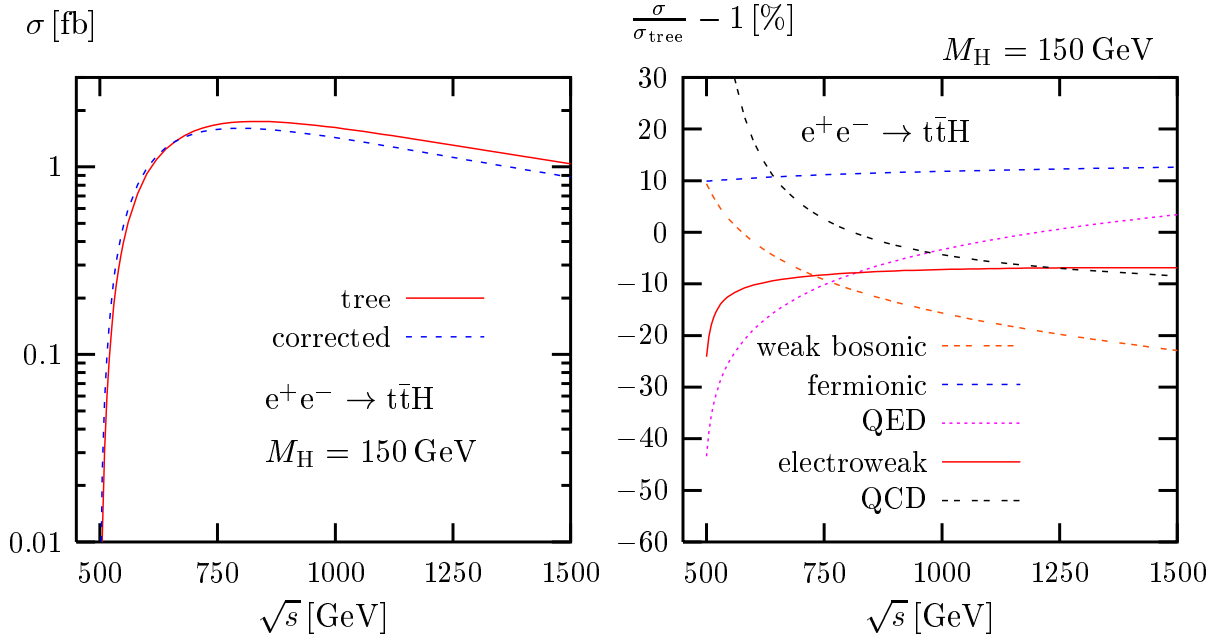
Another interesting Higgs production process is the reaction  $e^+e^- \rightarrow t\bar{t}H$ . If the Higgs-boson mass is not too large, i.e.  $M_H \sim 100$ – $200$  GeV, the Higgs boson is produced mainly through Higgs radiation off top quarks, while emission from intermediate Z bosons plays only a minor role. Therefore, this process can be used for the determination

of the top-quark Yukawa coupling,  $g_{t\bar{t}H}$ , which is by far the largest one in the SM ( $g_{t\bar{t}H} \approx 0.5$ ). If the Higgs boson is light, i.e.  $M_H \sim 120$  GeV, a precision of around 5% can be reached at an  $e^+e^-$  collider operating at  $\sqrt{s} = 800$  GeV with a luminosity of  $\int L dt \sim 1000 \text{ fb}^{-1}$  [17]. Combining the  $t\bar{t}H$  channel with information from other Higgs-production and decay processes an even better accuracy can be obtained in a combined fit [18].

The  $\mathcal{O}(\alpha_s)$  corrections to the total cross section within the SM have been calculated for the dominant photon-exchange channel in Ref. [19], while the full set of diagrams has been evaluated in Ref. [20]. Recently, considerable progress has been achieved in the calculation of the electroweak corrections to  $e^+e^- \rightarrow t\bar{t}H$ . Results for the electroweak  $\mathcal{O}(\alpha)$  corrections in the SM have been presented in Refs. [21,22,23]. While the results of Refs. [22,23] agree well, those of Ref. [21] differ at large CM energies and close to threshold.

In the last part of this article we show some results of our calculation of the  $\mathcal{O}(\alpha)$  electroweak corrections and of the  $\mathcal{O}(\alpha_s)$  corrections. Although the virtual corrections are much more complex than for  $e^+e^- \rightarrow \nu\bar{\nu}H$ , we were able to perform the calculation using the same computational techniques as in the former case. Results for total cross sections and various distributions have been presented in Ref. [23].

On the l.h.s. of Figure 2 we show the lowest-order cross section and the cross section including both electroweak and QCD corrections as a function of the CM energy for  $M_H = 150$  GeV. Away from the kinematic threshold at  $\sqrt{s} = 2m_t + M_H$  the total cross section is typically of the order of a few fb and becomes maximal at an energy of about 800 GeV. The relative corrections are presented on the r.h.s. of Figure 2. While the weak bosonic cor-



**Fig. 2.** Lowest-order and corrected cross section (l.h.s.) as well as relative corrections (r.h.s.) in the  $G_\mu$  scheme for a Higgs-boson mass  $M_H = 150$  GeV

rections are around +10% close to threshold and fall off rapidly with increasing CM energy, the fermionic corrections are about +10% and depend only weakly on the CM energy. For energies above 600 GeV, the fermionic and weak bosonic contributions partially cancel. The QED corrections, which include both the complete photonic and higher-order ISR corrections, are about -40% at threshold and rise to a few per cent at 1.5 TeV. In the electroweak corrections, both QED and weak contributions partially compensate each other. The QCD corrections are positive and rather large in the threshold region, where soft-gluon exchange between in the  $t\bar{t}$  system leads to a Coulomb-like singularity. In the region above threshold, the QCD corrections decrease and even turn negative for energies  $\gtrsim 800$  GeV.

In summary, for both processes  $e^+e^- \rightarrow \nu\bar{\nu}H$  and  $e^+e^- \rightarrow t\bar{t}H$  we find corrections that are typically of the order of  $\pm 10\%$  and thus important ingredients in the theoretical predictions for future  $e^+e^-$  colliders.

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